Absorption in Human Capital and R&D Effects in an Endogenous Growth Model*

Tiago Neves Sequeira†

Abstract

Until now, in models of endogenous growth with physical capital, human capital and R&D such as in Arnold [Journal of Macroeconomics 20 (1998)] and followers, steady-state growth is independent of innovation activities. We introduce absorption in human capital accumulation and describe the steady-state and transition of the model. We show that this new feature provides an effect of R&D in growth, consumption and welfare. We compare the quantitative effects of R&D productivity with the quantitative effects of Human Capital productivity in wealth and welfare.

JEL Classification: O15, O30, O41.

Key-Words: Human capital, Absorption, Endogenous Growth, Welfare.

*I am grateful to Ana Balcão Reis, José Tavares, António Antunes and Holger Strulik for their helpful insights on this paper. I also gratefully acknowledge financial support from the Fundação Amélia de Melo and POCI/FCT. The usual disclaimer applies.

†*Departamento de Gestão e Economia*, *Universidade da Beira Interior* and *INOVA, Faculdade de Economia*, *Universidade Nova de Lisboa.*

1 Introduction

This paper considers absorption of existing technologies by new human capital in a model with physical capital, human capital and R&D. The underline model follows Arnorld (1998, 2000) and Funke and Strulik (2000). In this literature, steady-state growth is not affected by innovative activities in the economy; but solely by human capital and preferences parameters. We show that the consideration of absorption also implies an effect of R&D productivity in economic growth and consumption. We access the quantitative effects of human capital and R&D productivity in growth and welfare.

Some contributions had focused on jointly considering Human Capital and R&D. The joint consideration of endogenous technology and human capital accumulation seems to have an important impact, as Barrio-Castro *et al.* (2002) and Zeng (2003) concluded within very different contexts.

We build on this literature to take into account the effect of absorption of technologies by new human capital. Zeng (2003) studied the impact of policies in the long-run growth in a model with R&D and human capital in which R&D policies also influence economic growth. He considered that human capital accumulation depended on both human and physical capital. However, he did not solve for the transition path of the economy. Furthermore, his mechanism was not the absorption of technologies by new human capital.

We do this both evaluating the quantitative effects of human capital and R&D productivities in the steady-state growth and calculating the whole transition path of a theoretical economy. King and Rebelo (1993) showed the importance of transition in explaining growth and development. We also add to the literature that deals with the effects of policies in the long-run (e.g. Peretto, 2003) as we use a model in which R&D directly influences long-run growth. For that, we improve on the human capital accumulation function, which was recognized by Funke and Strulik (2000:513) "as one of the most fruitful direction for future research". In fact, we use a human capital accumulation function that considers schooling and absorption of existing technologies as sources of human capital accumulation. This allows for important effects of R&D productivity in growth and welfare, in opposition to what happened with a simple Lucas (1988) function.

In Section 2, we present the model, we describe its transition dynamics and the steady-state. In Section 3, we describe the model quantitative properties and we quantitatively compare the effects of improving in education and R&D. Finally, we conclude in Section 4.

2 The model

The Model builds on Arnold (1998), who integrated human capital accumulation and R&D in the same model and studied the convergence properties of the model.[1] We add the consideration of absorption of new technologies by human capital.

2.1 Engines of Growth

2.1.1 The Human Capital Accumulation

Individuals may spend part of their human capital, H_H, on education. This non-market activity is described by a production function of the Uzawa (1965) - Lucas (1988) type. However, skills may also be accumulated through the contact to aggregated knowledge of the economy, which is seen as the absorption of the existing technologies by individual human capital. The following expression expresses these ideas

[1] The convergence properties of the model were recently re-assessed by Gómez (2005).

$$\dot{H} = \xi H_H + \gamma H^{\sigma} n^{1-\sigma}, \quad \xi, \gamma > 0; 0 < \sigma < 1. \tag{1}$$

where ξ is the productivity of schooling and it measures the incentive to spend time investing in human capital. This function interprets human capital accumulation as being dependent on schooling (ξH_H) and absorption ($\gamma H^{\sigma} n^{1-\sigma}$), being the first process only dependent on time dedicated to schooling (H_H) and absorption dependent on the stocks of individual human capital (H) and existing varieties on the economy (n); γ measures the relative importance of absorption in the human capital technology and σ measures the intensity of human capital needed to absorb the existing technological knowledge.[2] Absorption of human capital is seen here as a process of learning the existing technologies, which efficiency depends on the already accumulated human capital. This learning process contributes to the human capital in the economy.

Galor (2005) recognizes that "technology complements skills in the production of human capital", which is also the case in (1). Either (1996) argued that "the absorption of new technologies into production is skill-intensive". Contrary to this author, our absorption process is done in the human capital accumulation and not in the final production. We assume that human capital accumulates not only in school but also in contact with the stock of knowledge.

Jorgenson and Fraumeni (1992) calculated that human capital accounted for more that 95% of the USA education sector growth (1948-1986), which supports our assumption of just human capital in formal schooling. We assume the separability between schooling and absorption, which may be a simplifying assumption, but is essential to keep the model simulation tractable. Thinking in dynamic equilibrium however, we note that education has a positive effect on absorption: if individuals devote more time to schooling, human capital increases more, which

[2] Making $\gamma = 0$ transform this function into the Uzawa (1965) - Lucas (1988) framework.

move individuals' human capital upwards and thus promote absorption of new technologies.[3]

It is well-known from previous contributions (e.g. Arnorld, 1998, Funke and Strulik, 2000), which considered $\gamma = 0$, that there is no effect of R&D productivity in the steady-state in this type of model. Zeng (2003) noted this problem and solved it by considering a Cobb-Douglas function for accumulation of human capital that included both human and physical capital as production factors. This implied that R&D influences long-run growth, because as human capital was also produced with physical capital, the rate of return (r) would be dependent not only on the human capital productivity but also on the investments rate of return. Thus, by arbitrage conditions and the fact that physical capital is an input to R&D in his article, the interest rate will be dependent on R&D parameters. We differ from the Zeng (2003) contribution in three main aspects: (1) our human capital accumulation function assumes that human capital is produced by human capital (schooling and total) and varieties; (2) we focus on all transition path of the economy and (3) we focus on welfare rises due to Human Capital or R&D.

2.1.2 The Production of new Ideas

Production of a new intermediate good requires the invention of a new blueprint. We assume that output of new ideas is determined solely by the aggregate knowledge. The production of new ideas is made according to:

$$\dot{n} = \epsilon H_n, \quad \epsilon > 0 \tag{2}$$

where H_n is human capital allocated to R&D activities and ϵ is the productivity of R&D.

[3] Absorption can be interpreted as an activity done in entrepeurnial activities. Iyigun and Owen (1999) considered the existence of separate production functions for professional and entrepreneurial human capital. Moreover, both human capital types contributes to the R&D process, which is also what happens here. We exclude the existence of human capital depreciation, as previous contributions also did, as this does not influence our results.

Let υ_t denote the expected value of innovation, defined by

$$\upsilon_t = \int_t^{\infty} e^{-[R(\tau)-R(t)]}\pi(\tau)d\tau, \ where\ R(t) = \int_0^t r(\tau)d\tau \tag{3}$$

Taking into account the cost of innovation as implied by (2), free entry conditions in R&D are defined as follows:

$$w/\epsilon = \upsilon \text{ if } \dot{n} > 0 \ (H_n > 0) \text{ or} \tag{4}$$

$$w/\epsilon > \upsilon \text{ if } \dot{n} = 0 \ (H_n = 0). \tag{5}$$

where w is the wage paid to human capital.

Finally, no-arbitrage requires that the valorization of the patent plus profits is equal to investing resources in the riskless asset:

$$\dot{\upsilon} + \pi = r\upsilon \Leftrightarrow \frac{\dot{\upsilon}}{\upsilon} = r - \pi/\upsilon. \tag{6}$$

2.2 Production technologies and market structure

The output of the final good depends on the physical capital (K), human capital allocated to final good production (H_Y) and differentiated goods (D), using a Cobb-Douglas technology:

$$Y = A_1 K^{\beta} D^{\eta} H_Y^{1-\beta-\eta}, \ \beta, \ \eta > 0, \ \beta + \eta < 1 \tag{7}$$

The index of intermediates is represented by the usual Dixit and Stiglitz formulation:

$$D = \left[\int_0^n x_i^{\alpha} di \right]^{1/\alpha}, \quad \alpha < 1 \tag{8}$$

where n denotes the number of available varieties and x_i is the quantity of the intermediate good i that is produced with the final good, in a one-to-one proportion. The elasticity of substitution between varieties is $\varepsilon_x = 1/(1-\alpha) > 1$. Physical capital is used only for the production of final goods. For simplicity, we neglect physical capital depreciation, which leads to the economy resource constraint:

$$Y = C + \dot{K} + \int_0^n x_i di \tag{9}$$

Markets for the final good and its factors are perfectly competitive and the final good price is normalized to one. Profit maximization, taking the interest rate (r), the aggregated price of the differentiated good (P_D) and the wage (w) as given, implies the following inverse-demand functions:

$$r = \frac{\beta Y}{K}, \tag{10}$$

$$P_D = \frac{\eta Y}{D}, \tag{11}$$

and

$$w = \frac{(1-\beta-\eta)Y}{H_Y}. \tag{12}$$

Each firm in the differentiated-goods sector owns a patent for selling its variety x_i. Producers act under monopolistic competition and maximize operating profits

$$\pi_i = (P_{x_i} - 1)x_i \tag{13}$$

The variable P_{x_i} denotes the price of an intermediate and 1 is the unit cost of Y. From profit maximization in the intermediate-goods sector, each firm charges a price

$$P_{x_i} = 1/\alpha \tag{14}$$

With identical technologies and symmetric demand, the quantity supplied is the same for all goods, $x_i = x$. Hence, equation (8) simplifies to

$$D = n^{1/\alpha}x \tag{15}$$

From $P_D D = pxn$ together with equations (14) and (15) we obtain the total quantity of intermediates employed as

$$X = xn = \alpha\eta Y \tag{16}$$

After insertion of equations (14) and (16) into (13), profits can be rewritten as a function of aggregate output and the number of existing firms:

$$\pi = (1 - \alpha)\eta Y/n \tag{17}$$

Before we proceed with the analysis we compute some equations that will be useful. Insertion of equation (16) in equation (9) simplifies the resource constraint to

$$\dot{K} = (1 - \alpha\eta)Y - C \tag{18}$$

and insertion of (15) and (16) in the production function (7) gives (after time-differentiation) the output growth rate:

$$(1 - \eta)g_Y = \beta g_K + [\frac{1-\alpha}{\alpha}]\eta g_n + (1 - \beta - \eta)(g_{u_1} + g_H) \tag{19}$$

where $u_1 = H_Y/H$ is the proportion of knowledge allocated to final good production and where the growth rate of variable z is denoted by g_z. Log-differentiation of equations (10) and (12) provides

$$g_r = g_Y - g_K \tag{20}$$

$$g_w = g_Y - (g_{u_1} + g_H) \tag{21}$$

2.3 Households

Each individual allocates his knowledge between the different activities in the economy, such that:

$$H = H_H + H_n + H_Y \tag{22}$$

Individuals earn wages, w, per unit of employed labor $(H - H_H)$ and returns, r, per unit of individual wealth. They maximize intertemporal utility

$$U_t = \int_t^{\infty} \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho(\tau - t)} d\tau \tag{23}$$

(where $\rho > 0$ denotes the time preference rate and θ is the coefficient of relative risk aversion), subject to $\dot{a} = w(H - H_H) + ra - C$ and to eq. (1).[4] Using the control variables $C > 0$ and $H_H \geq 0$ and the state variables a and H, we write the current value Hamiltonian

$$\Xi = \frac{C_t^{1-\theta} - 1}{1-\theta} + \lambda_1 (w(H - H_H) + ra - C) + \lambda_2 (\dot{H}) \tag{24}$$

where $\dot{H}$ is given by (1) in the second restriction. We obtain from its first order conditions, the following expressions for consumption and wage growth rates:

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\theta} \tag{25}$$

$$H_H > 0 \text{ and } \frac{\dot{w}}{w} = (r - \xi - \sigma\gamma \left(\frac{1}{H/n}\right)^{1-\sigma}) \text{ or } H_H = 0 \tag{26}$$

Equation (25) is the standard Ramsey rule. Equation (26) indicates that the growth rate of wages must be sufficiently high compared to the interest rate to ensure investment in human capital.

In the following section we describe the dynamics of the model and its steady-state.

[4] Although individuals have finite lives, we consider an immortal extended family that makes intergenerational transfers based on altruism (see Barro and Sala-i-Martin (1995:60)). We solve the consumer utility problem assuming that the absorption process is dependent not on individual human capital but on individual human capital.

2.4 Dynamics and Steady-State

For an innovative economy, eq. (4) must hold. Using eq. (4), equation (6) can be re-written as

$$g_w = r - \epsilon\pi/w \tag{27}$$

After substitution of profits from eq.(17), wages from eq.(12) and the growth rate of wages from eq.(26) into equation (27), we obtain the human capital share in final good production, u_1:

$$u_1 = \frac{1}{\epsilon}\frac{(1-\beta-\eta)}{(1-\alpha)\eta}\left[\xi + \sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}\right]\frac{n}{H} \tag{28}$$

From this equation and eq.(21) the growth rate of innovations can be written as

$$g_n = g_Y - g_w + g_{H/n}(1-\sigma)\frac{\sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}}{\left[\xi + \sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}\right]} \tag{29}$$

Insertion of eqs.(20) and (21) into eq.(19) provides the growth rate of the interest rate according to:

$$g_r = -\frac{1-\beta-\eta}{\beta}g_w + \frac{1-\alpha}{\alpha}\frac{\eta}{\beta}g_n \tag{30}$$

We define the knowledge-ideas ratio as H/n and obtain from equations (1), (2), and (28) its dynamics:

$$g_{H/n} = \gamma\left(\frac{1}{H/n}\right)^{1-\sigma} + \xi\left[1 - \left(\left(\xi + \sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}\right)\frac{(1-\beta-\eta)}{(1-\alpha)\eta}\frac{1}{\epsilon} + \frac{1}{\epsilon}g_n\right)(n/H)\right] - g_n \tag{31}$$

Inserting (31) into (29) and using (10), (18) and (26), we reach:

$$g_n = \frac{1}{1 - \frac{1-\alpha}{\alpha}\frac{\eta}{\beta} + B_1 B_2}\left[\frac{1-\alpha\eta}{\beta}r - C/K - \frac{1-\eta}{\beta}\left(r - \xi - \sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}\right) + B_1 B_3\right], \tag{32}$$

where $B_1 = (1-\sigma)\frac{\sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}}{\left[\xi + \sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}\right]}$; $B_2 = \left[\frac{\xi/\epsilon}{H/n} + 1\right]$ and

$$B_3 = \gamma\left(\frac{1}{H/n}\right)^{1-\sigma} + \xi\left[1 - \left(\left(\xi + \sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}\right)\frac{(1-\beta-\eta)}{(1-\alpha)\eta}\frac{1}{\epsilon}\right)(n/H)\right].$$

Inserting (26) and (32) into (30), we reach a new equation for g_r :

$$\begin{aligned} g_r = -\frac{1-\beta-\eta}{\beta}\left(r - \xi - \sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}\right) + \\ +\frac{1-\alpha}{\alpha}\frac{\eta}{\beta}\frac{1}{1 - \frac{1-\alpha}{\alpha}\frac{\eta}{\beta} + B_1 B_2}\left[\frac{1-\alpha\eta}{\beta}r - C/K - \frac{1-\eta}{\beta}\left(r - \xi - \sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}\right) + B_1 B_3\right] \end{aligned} \tag{33}$$

Finally, from the definition of C/K, using (10), (18) and (25):

$$g_{C/K} = \left(1/\theta - \frac{1-\alpha\eta}{\beta}\right)r + C/K - \rho/\theta \tag{34}$$

The dynamics of the model can be characterized by (31), (33) and (34). These are the equations that we integrate by the backward integration method. By (18) and (25), in the steady-state, $g_Y^* = g_C^* = \frac{r^*-\rho}{\theta}$

The Proposition 1 derives the steady-state expressions for the model.

Proposition 1 *Let $\xi > \rho$ and $\theta > 1$. There is one positive steady-state of the model given by $(r^*, (C/K)^*, (H/n)^*)$:*

$$r^* = \frac{\theta\left(\xi + \sigma\gamma\left(\frac{1}{H/n}\right)^{1-\sigma}\right)(1+A_2) - \rho}{(\theta-1)+\theta A_2} \tag{35}$$

$$C/K^* = \left(\frac{1-\alpha\eta}{\beta} - 1/\theta\right) r^* + \rho/\theta \tag{36}$$

where $A_2 = \frac{1-\eta-\beta}{\eta}\frac{\alpha}{1-\alpha}$. The expression for the steady-state value of H/n^* is obtained equating (31) to zero and solving for $(H/n)^*$. This yields that $(H/n)^*$ is a root of the following polynomial in Z :

$$\xi\sigma\gamma(1-\beta-\eta)Z^{2-\sigma} + \gamma\eta\epsilon(1-\alpha)Z^{1-\sigma} - \xi^2(1-\beta-\eta)Z \tag{37}$$

$$-(\xi+\epsilon)\eta g_n^* + \xi\eta\epsilon(1-\alpha) = 0$$

where

$$g_n^* = \frac{1}{1 - \frac{1-\alpha}{\alpha}\frac{\eta}{\beta} + \frac{\xi/\epsilon}{(H/n)^*}B_1}\left[\frac{1-\alpha\eta}{\beta}r^* - (C/K)^* - \frac{1-\eta}{\beta}\left(r^* - \xi - \sigma\gamma\left(\frac{1}{(H/n)^*}\right)^{1-\sigma}\right)\right] \tag{38}$$

Proof. We obtain (35) to (37) equating (31), (34), and (33) to zero. Last equation is obtained using (32) and the fact that in steady-state $B_3 = g_n^*$, using (31). For our choice of parameters, there is only one real positive root of (37), which guarantees that the steady-state exists and is unique. ■

In the Appendix, we derive the Jacobian of this system and show that for our choice of parameters the system converges along a two-dimensional stable manifold to a unique steady-state.

2.4.1 Discussion

With $\gamma = 0$ the R&D productivity (and then any policy that influences it) does not influence growth rates.[5] This happens because in this model agents can re-allocate their human capital effort between three different uses: final good, human capital accumulation and research. When R&D productivity decreases, people allocate more effort to other activities than research. This implies that it is not affecting growth rates at the steady-state but only allocation of resources through sectors. This is the typical result according to which R&D policies do not influence steady-state growth rates (see e.g. Arnold, 1998 and Funke and Strulik, 2000). This fact indicates that we should expect a low impact of ϵ in explaining differences of output, consumption and welfare if $\gamma = 0$. In the Absorption Model presented above ($\gamma > 0$), this mechanism continues to happen. However, as a fall in productivity of R&D also decreases the productivity of human capital in the absorption process, the long-run growth falls. It should be noted that effects in ξ and ϵ can be seen as induced by revenue-neutral subsidies to education and R&D. A subsidy to education would increase ξ and a subsidy to R&D would increase ϵ, both in the

[5]To see this, make $\gamma = 0$ in eq.(35) and note that $g_Y^* = \frac{r^* - \rho}{\theta}$.

same amount $1/(1 - subsidy)$.[6] Thus, to keep the analysis simpler, we concentrate on effects in productivities.

We proceed by backward integration (Brunner and Strulik, 2002) and integrate the model. We begin arbitrarily close to the steady state and we backward integrate equations that describe the evolution of r (33), the evolution of C/K (34), and the evolution of H/n (31), until we reach given values for r_0 and H/n_0.[7]

3 Calibration and Results

3.1 Calibration

Parameters for our exercises were mainly taken from Gómez (2005). The additional parameters are the weight of absorption in the human capital accumulation function (γ) and the share of human capital in the absorption parcel of the human capital accumulation function (σ). As the human capital accumulation technology is different from previous contributions, we also calibrate the productivity of schooling (ξ). We calculate ξ and γ to replicate the *per capita* average growth rate of GDP in the USA. We choose to replicate a rate of 2.102%, which represents the evolution of GDP *per capita* from 1948 and 1986, reported by Maddison (1995). As a first exercise we assume that $\gamma = 0$ and consider a schooling productivity ξ that replicated the mentioned rate. We call this a "Lucas" exercise. Then, we assume $\gamma = 0.025$ and again calculate ξ to replicate the output growth rate of 2.102%. We assume a value of $\sigma = 0.5$, which means equal shares of human capital and varieties in determining absorption. We have

[6] The implementation of a subsidy of 5% to education or to R&D is similar to a 5.26% increase in the respective productivity.

[7] We employ a fourth-order Runge-Kutta method with variable step control provided by Matlab. We applied a maximum discretization error of 10^{-11}. Matlab codes are available upon request.

tested values from 0.95 to 0.05, which do not change our main result.[8] We call this exercise the Absorption exercise.

The next table summarizes parameters for the calibration.

Table 1: Calibration Values

Parameters	"Lucas"	"Absorption"
α	0.40	0.40
β	0.36	0.36
η	0.36	0.36
ϵ	0.1	0.1
ξ	0.051198	0.0294
ρ	0.023	0.023
θ	2	2
σ	–	0.5
γ	0	0.025

3.2 The Influence of R&D and Human Capital in the Steady-State

Here, we show some implications of variations in ξ and ϵ, that represent an increase in incentives to invest in R&D and to accumulate human capital, respectively, in the steady-state.

For ease of comparison, we state results on 1% and 5% rises in the initial values of ξ and ϵ. Table 2 summarizes the results.

Table 2: Steady-State Implications

	Benchmark	Rise in Productivity			
		1%		5%	
	Values	H	R&D	H	R&D
ξ/ϵ		ξ	ϵ	ξ	ϵ
		Lucas ($\gamma = 0$)			
g_Y^*	2.102%	2.140%	2.102%	2.293%	2.102%
H/n^*	0.855	0.866	0.847	0.907	0.815
		Absorption ($\gamma = 0.025$)			
g_Y^*	2.102%	2.113%	2.111%	2.158%	2.145%
H/n^*	0.328	0.333	0.325	0.351	0.312

[8] With $\sigma = 1$ the impact of R&D in growth and welfare would be near null, because n would not influence human capital accumulation. With $\sigma = 0$ the impact of R&D in growth and welfare would also be near null, because the agent does not decide the number of varieties n, which is in the human capital accumulation function. This implies that n does not determine wages nor growth.

The table shows that the consideration of absorption implies an impact of R&D productivity in growth, as with the model without absorption, the growth rate remains equal between the benchmark case and both cases with rises in ϵ (2.102%). However, in the model with absorption the effect of a change in R&D productivity is positive, being almost equal to the effect of a change in human capital productivity. Increases in human capital accumulation productivity naturally imply a rise in the equilibrium human capital to varieties ratio and increases in R&D productivity imply a fall in the steady-state human capital to varieties ratio.

In the next section, we consider welfare effects taking all the transition dynamics into account. The effect on welfare of increasing productivities in human capital accumulation or R&D cannot be directly driven from the steady-state effects as transitional effects may also influence welfare and production.

3.3 The Influence of R&D and Human Capital in wealth and welfare taking Transition into account

In order to present results that take in account the evolution of an economy with absorption, we proceed as Brunner and Strulik (2002). First we backward integrate the three differential equations that describe the transition applying the benchmark calibration described in the table 1. Then, we compare these results with four different exercises: a 1% rise in human capital accumulation productivity (ξ); a 1% rise in R&D productivity (ϵ); a 5% rise in human capital accumulation productivity (ξ) and a 5% rise in R&D productivity (ϵ).[9] In all exercises, we approximate the real interest rate (r_0) and the human capital to varieties ratio (H/n_0) from the initial values obtained in the benchmark exercise, as these are predetermined variables.[10]

[9] A 5% rise in produtivities is equivalent to a new subsidy of 4.68%.

[10] For a discussion on this, see Gómez (2005). Consequently, we also consider that the initial value for C can jump across simulations (although K remains fixed).

We first describe the transition path of most important variables in the "Lucas" model and in the "Absorption" model. Then, we compare the impact of different productivities in output, consumption and utility. The following figures describe the evolution of the "Lucas" economy in the first 180 years.

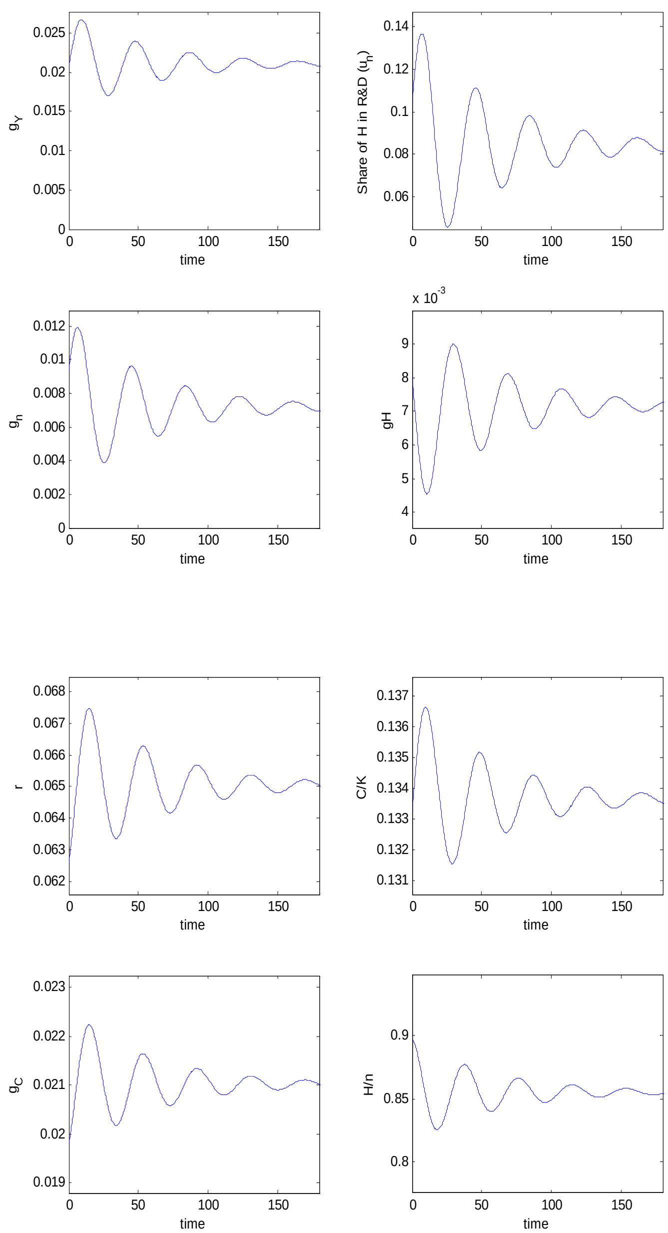

Figure 1: Transition Paths for Representative Variables in the "Lucas" Calibration

The figure shows an oscillatory pattern of convergence as in Gómez (2005) and a lengthy transition to the steady-state: the steady-state is not reached before 580 years.[11] Next figures present the transition path of the absorption economy.

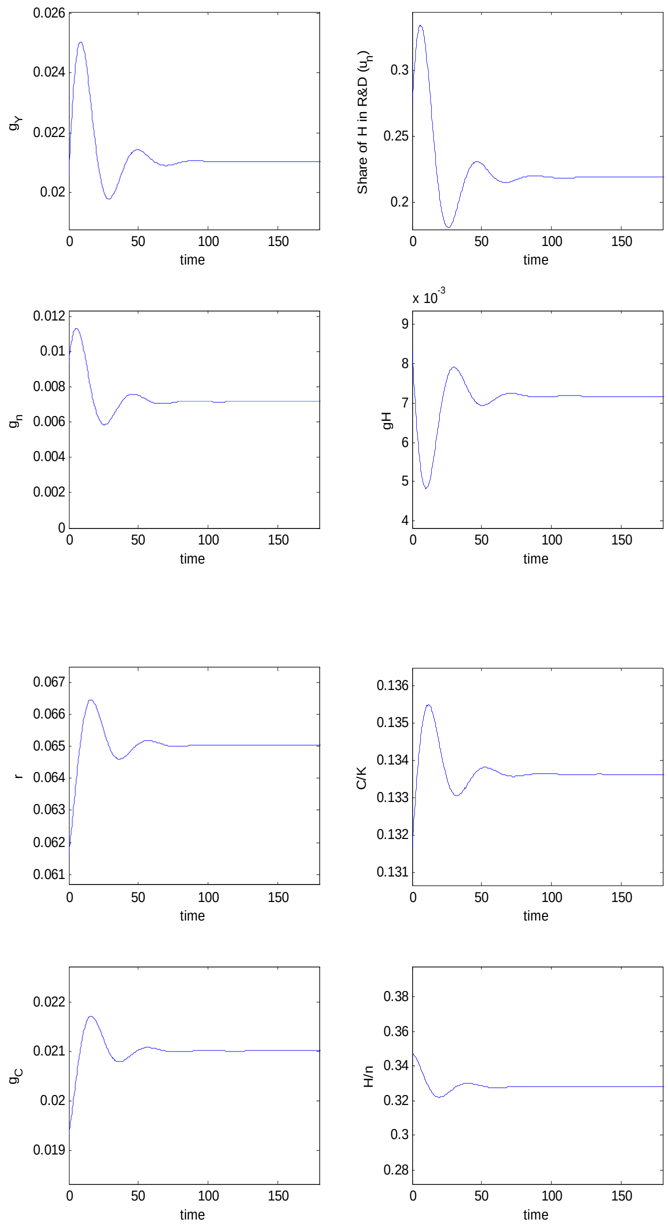

Figure 2: Transition Paths for Representative Variables in the "Absorption" Calibration

[11] Rigorously, the economy only approaches steady-state. We consider that steady-state is reached if the three variables that describe the reduced form of the model are stable at a 6 digit approximation.

The figure shows that the economy with absorption presents a shorter transition path, reaching the steady-state near 150 years after the beginning. In this feature, this economy seems to be more close to reality that the simpler "Lucas" economy. The economy maintains the oscillatory pattern which can be characterized by an initial overshooting of the final values for most variables.

Now, we want to compare the welfare effects of rising productivities in both models, in order to demonstrate our claim according to which the model with absorption shows higher effects of R&D policies than the simpler model. First we present two figures that show the evolution of consumption in the first 200 years after the beginning (Figure 3). In order to keep the figures clear, we only compare the benchmark case with the 5% rise in both parameters.

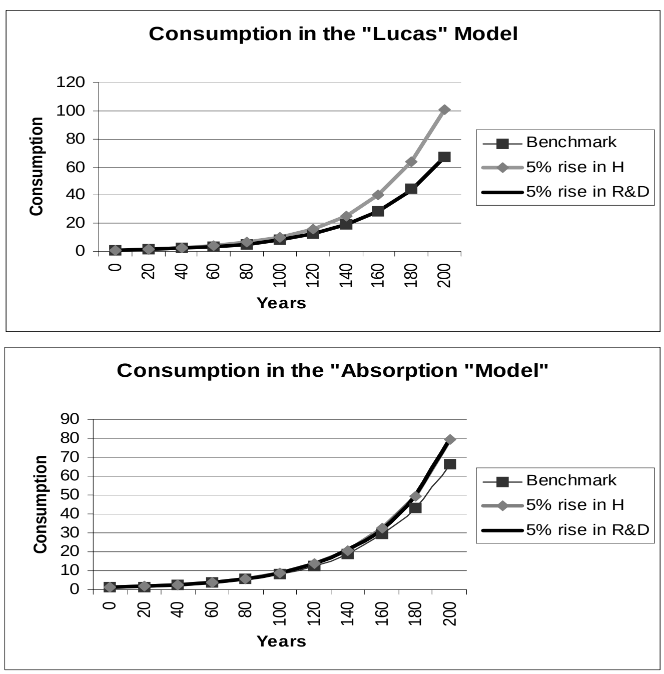

Figure 3: Consumption Paths in the "Lucas" and "Absorption" Calibration

The first figure (The "Lucas" Model) shows that while increasing the human capital accumulation productivity increase the consumption path, increasing the R&D productivity keep the consumption path almost equal to that with the initial value for research productivity. The second figure shows that both human capital productivity rises and R&D productivity rises move upward the consumption path. In fact, consumption paths of both human capital and R&D rises are almost the same.

Next table shows values for output, consumption (calculated after 200 years) and utility for each experiment.[12]

Table 3: Output and Welfare Implications

	Benchmark	Rise in Productivity			
		1%		5%	
	Values	H	R&D	H	R&D
ξ/ϵ		ξ	ϵ	ξ	ϵ
Lucas ($\gamma = 0$)					
Y	171139	187980	172920	259061	170078
C	67.20	73.67	68.10	100.7	66.89
U	64.30	64.80	64.50	65.72	64.15
$\Delta Y/Y^{bench}$	–	8.22%	0.43%	51.89%	-0.70%
$\Delta C/C^{bench}$	–	9.63%	1.34%	49.90%	-0.46%
$\Delta U/U^{bench}$	–	0.79%	0.31%	2.22%	-0.23%
Absorption ($\gamma = 0.025$)					
Y	170883	183310	180967	194757	196342
C	66.25	74.68	73.63	79.25	79.95
U	63.44	64.56	64.33	64.15	64.81
$\Delta Y/Y^{bench}$	–	7.27%	5.90%	13.97%	14.90%
$\Delta C/C^{bench}$	–	12.72%	11.14%	19.62%	20.68%
$\Delta U/U^{bench}$	–	1.77%	1.41%	1.11%	2.16%

From the analysis of the table, we confirm that in the "Lucas" framework only human capital productivity (policy) has significative effects in output, consumption and in utility. However, in the "Absorption" model R&D productivity (policy) becomes as important or even more important (in the 5% rise case) than human capital productivity (policy).

[12] According to (23), utility is calculated as $U_{ss} = \frac{\frac{C_{ss}^{1-\theta}-1}{1-\theta}+\frac{g_C}{\rho}}{\rho-(1-\theta)g_C}$. As in the Gómez (2005) model, output is predetermined. As an initial value, we have considered the USA output in 1870 (2457 dolars).

4 Conclusions

We add to Arnold (1998, 2000) the consideration of absorption as a source of human capital accumulation. We present the steady-state of the model, as well as simulated its transition along a balanced growth path. This allowed for a dramatic increase in the effect of R&D when compared to a more usual Lucas-type human capital accumulation. This article complements that of Zeng (2003) as we calculate the complete transition path for the economy and focus on output, consumption and welfare and not only on growth rates. Thus, these conclusions indicate the relevance of future empirical research on the relative importance of absorption by human capital accumulation in different economies.

References

[1] Arnold, L. (1998), "Growth, Welfare and Trade in an Integrated Model of Human-Capital Accumulation and Research", *Journal of Macroeconomics,* 20(1), 81-105.

[2] Arnold, L. (2000), "Endogenous growth with physical capital, human capital and product variety: A Comment", *European Economic Review*, 44, 1599-1605.

[3] Barrio-Castro, T., E. López-Bazo, and G. Serrano-Domingo (2002), "New Evidence on International R&D spillovers, human capital and productivity in the OECD", *Economics Letters*, 77, 41-45.

[4] Brunner, M. and H. Strulik (2002), "Solution of Perfect Foresight Saddlepoint Problems: A simple method and Aplications", *Journal of Economics Dynamics and Control*, 25(5):737-753, May.

[5] Eicher, T. (1996), “Interaction Between Endogenous Human Capital and Technological Change”, *Review of Economic Studies*, 63, 127-144.

[6] Funke, M. and H. Strulik (2000), “On Endogenous growth with physical capital, human capital and product variety”, *European Economic Review*, 44, 491-515.

[7] Gómez, M. (2005), “Transitional Dynamics in an Endogenous Growth Model with Physical Capital, Human Capital and R&D”, *Studies in Nonlinear Dynamics and Econometrics,* Vol. 9(1), Article 5. http://www.bepress.com/snde/vol9/iss1/art5.

[8] Iyigun, M. and Owen, A. (1999), “Entrepreneurs, Professionals and Growth”, *Journal of Economic Growth*, vol. 4, number 2, 215-232.

[9] King, R. and Rebelo, S. (1993), “Transitional Dynamics and Economic Growth in the Neoclassical Model”, *American Economic Review*, 83(4), September, 908-931.

[10] Jorgenson, D. and B. Fraumeni (1993), “Education and Productivity Growth in a Market Economy”, *Atlantic Economic Journal,* vol. 21, issue 2, June, 1-25.

[11] Lucas, R. (1988), “On the Mechanics of Economic Development”, *Journal of Monetary Economics*, 22, 3-42.

[12] Maddison, A. (1995), *Monitoring the World Economy 1820-1992,* Development Center Studies, OECD, Paris.

[13] Peretto, P. (2003), “Fiscal Policy and Long-Run Growth in R&D based Models with endogenous Market Structure”, *Journal of Economic Growth*, v.8, June, 325-347.

[14] Uzawa, H. (1965), “Optimal Technical Change in an Aggregative Model of Economic Growth”, *International Economic Review* 6*, 18-31.*

[15] Zeng, J. (2003), "Reexamining the interaction between innovation and capital accumulation", *Journal of Macroeconomics* 25, *541-560.*

A Appendix: Jacobian of the Linearized Systems

Linearizing the system (34), (33) and (31) around its steady-state $(r^*, C/K^*, H/n^*)$, the dynamics can be approximated by the following third order system:

$$\begin{pmatrix} \dot{r} \\ \dot{C/K} \\ \dot{H/n} \end{pmatrix} = \begin{pmatrix} \left(\frac{\alpha\eta(1-\alpha)}{\alpha\beta-(1-\alpha)\eta} - \frac{1-\beta-\alpha\eta}{\beta}\right) r^* & -\frac{\eta(1-\alpha)}{\alpha\beta-(1-\alpha)\eta} r^* & \Omega_1 \\ (1/\theta - (1-\alpha\eta)/\beta)\, C/K^* & C/K^* & 0 \\ \Omega_2 & \Omega_3 & \Omega_4 \end{pmatrix} \begin{pmatrix} r - r^* \\ C/K - C/K^* \\ H/n - H/n^* \end{pmatrix} \quad (39)$$

where

$$\Omega_1 = (\sigma-1)(\frac{H}{n})^{\sigma-2}\sigma\gamma\frac{\alpha(1-\beta-\eta)+(1-\alpha)\eta}{\alpha\beta-(1-\alpha)\eta}r^*; \quad (40)$$

$$\Omega_2 = \frac{1}{1-\frac{1-\alpha}{\alpha}\frac{\eta}{\beta}+B_1B_2}\left[\frac{1-\alpha\eta}{\beta}\left(\frac{H}{n}\right)^*\right]; \quad (41)$$

$$\Omega_3 = -\frac{1}{1-\frac{1-\alpha}{\alpha}\frac{\eta}{\beta}+B_1B_2}\left[\frac{1-\alpha\eta}{\beta}\left(\frac{H}{n}\right)^*\right]; \quad (42)$$

$$\begin{aligned}\Omega_4 = {} & (\sigma-1)\gamma\left((\frac{H}{n})^*\right)^{(\sigma-1)} + \xi(\xi+p)\frac{\epsilon(1-\beta-\eta)}{(1-\alpha)\eta}\left(\frac{H}{n}\right)^* - \\ & -\xi\frac{\epsilon(1-\beta-\eta)}{(1-\alpha)\eta}\sigma\gamma(\sigma-2)\left((\frac{H}{n})^*\right)^{(\sigma-2)} + g_n^*\left(\frac{H}{n}\right)^*\frac{1}{\epsilon}. \end{aligned} \quad (43)$$

We are now able to calculate the eigenvalues for each one of the presented exercises. As we have 2 predetermined variables we need two stable roots. In the "Absorption" Model - that corresponds to the calibration in column (2) in Table 1, the real parts of the eigenvalues are -0.0121, -0.0121 and 0.0267. Values for other exercises are available upon request. For ease of comparison in a model with $\gamma = 0$ (e.g. in Gómez (2005)), $\Omega_1 = \Omega_3 = \Omega_4 = 0$; $\Omega_2 = \xi - g_n^*$.